p-Value Cumulative Sum (CUSUM) Chart: A Tool for Monitoring Inflation Rate

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Received: November 18, 2015; Revised: March 3, 2016; Accepted: March 23, 2016

Abstract

This study is aimed at analyzing the rates of Inflation and designing p-Value Cumulative Sum (CUSUM) chart. The data used in this study is a secondary data on Inflation rates in Nigeria (January, 2002 to December, 2011) obtained from the Library of Central Bank of Nigeria (CBN), Ilorin, Kwara State. Analysis on the Inflation rates in Nigeria recorded during the years of study (2002-2011) was carried out using Cumulative Sum (CUSUM) and p-value Cumulative Sum (p-CUSUM) control chart schemes. A current statistical software (R) was used to analyze and plot the charts in this study. The result obtained from the analysis revealed that the p-value CUSUM detected a shift at sample number 65, while the ordinary CUSUM chart defected at point 67. This makes the p-CUSUM control chart a better statistical tool in detecting small shift in the process level than ordinary CUSUM control chart scheme.

Keywords: Average Run Length (ARL), Consumer Price Index (CPI), Upper Control Limits (UCL), Target Value, Reference Value

1.0 Introduction

The maintenance of price stability is one of the macroeconomic challenges facing the Nigerian government in our economic history. This elusive factor is known and referred to as inflation and it is defined by economists as "a continuous rise in prices". By definition, inflation is a persistent and appreciable rise in the general level of prices [1]. Not every rise in price level is termed inflation. Therefore, for a rise in the general price level to be considered inflation, such a rise must be constant, enduring and sustained. The rise in price should affect almost every commodity and should not be temporal. Some researchers are more explicit, referring to inflation as a continuing rise in prices as measured by an index such as the Consumer Price Index (CPI) or by the implicit price deflator for Gross National Product [1].

In an inflationary economy, it is difficult for the national currency to act as medium of exchange and a store of value without having an adverse effect on income distribution, output and employment [2]. Inflation is characterized by a fall in the value of the country's currency and a rise in her exchange rate with other nation's currencies. This is quite obvious in the case of the value of the Naira (₦), which was ₦ 1 to $1 (one US Dollar) in 1981, average of ₦ 100 to $1 in year 2000 and over ₦ 128 to $1 in 2003, [3]. This decline in the value of the Naira coincides with the period of inflationary growth in Nigeria, and is an unwholesome development that has led to a drastic decline in the living standard of the average Nigerian. There are three approaches to measure inflation namely, Gross National Product (GNP), Consumer Price Index (CPI) and Wholesome or Producer Price Index (WPI or PPI). The period to period changes in these two latter approaches (CPI and WPI) are regarded as direct measures of inflation. There is no single one of the three that rather uniquely best measure inflation. The CPI approach, though the least efficient of the three, is used to measure inflation rates in Nigeria as it is easily and currently available on monthly, quarterly and annual basis [4]. Existence of excess aggregate demand can cause inflation (demand pull inflation). Cost-push inflation arises from upward pressure of production costs, while structural inflation arises from constraints such as inefficient production, marketing and distribution systems in the productive sectors of the economy.

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Inflation has been apparent in Nigeria from the outset of our national life. This was propelled in the 1960s through the "cheap money policy" adopted by the government to stimulate development after independence. Interest rates were lowered and targeted at the preferred sectors of the economy, and was meant to facilitate the implementation of the First National Development Plan and subsequently the prosecution of the civil war. This led to rapid monetary expansion with the narrow and broad measures of money stock rising at annual rates of 29.7% (1961) and 44% (1969). Consequently, inflation increased from 6.4% (1961) to 12.1% (1969).

Since mid 1960s, inflation has become so serious and contentious a problem in Nigeria. Though inflation rate is not new in the Nigerian economic history, the recent rates of inflation have been a cause of great concern to many. During the period (1981–2003), there has been an upsurge in the inflationary rates leading to major economic distortions. This study is aimed at designing and comparing ordinary CUSUM and p-Value CUSUM control charts. These techniques are used to monitor the rates of inflation of goods and services so that an earlier attention to curb the occurrence of inflation rates can be given. More so, these applications may have uses well beyond the banking industries.

2.0 Materials and Methods

The data used in this study is a secondary data on Inflation rates in Nigeria extracted from the Statistical Journal (2011) from the Library of Central Bank of Nigeria (CBN), Ilorin, Kwara State Branch for a period of ten years (2002-2011) which consist of the Consumer Price Index (CPI), month on change in percent (%) and year on change in percent (%). The role of Central bank of Nigeria is to put the appropriate strategy in place aimed at reducing the inflation rates in the country whenever it is detected. Such detection can be facilitated by the use of control chart schemes.

2.1 Cumulative Sum (CUSUM) Approach

The Cumulative Sum Charts was introduced by Page [5] and was later developed by many authors and proposed as an alternative to Shewart Charts [6, 7]. These authors directly incorporated all of the information in the sequence of the sample values and detect small shifts in the process level more quickly. The chart plots the cumulative sums of the deviations from a target value using samples from all prior observations against sample numbers. Control charts are more meaningful graphically, as process shifts are often easy to detect and points of change can also be easily located. The CUSUM control chart has a rather long memory due to the fact that it uses a non-weighted sum of all prior observations.
Suppose a total of N samples are processed, let x_i be i^{th} observation. The N^{th} CUSUM (S_N) is defined to be the sum of the deviations of the first i^{th} observations from the process aim value [8]. Using the sample mean $\bar{X}$ as the preliminary aim, the N^{th} CUSUM is $S_N = \sum_{i=1}^{N}(X_i - \overline{X})$.
Let max (a, b) be the maximum of a and b. The i^{th} CUSUM for detecting an upward shift (S_i) for the i^{th} observation is defined as;
$S_i = \max(0, X_i - k_1 + S_{i-1})$; where k_1 is the reference value of the ''upward'' CUSUM.
The value of k is chosen relative to the size of the shift to be detected and S_0 is the head start value for the upward shift (usually set to 0). Similarly, the i^{th} CUSUM for detecting a downward shift (S_i) for the i^{th} observation is $S_i = \min(0, X_i - k_2 + S_{i-1})$; where k_2 is the reference value of the ''downward'' CUSUM. S_0 is the head start value for the downward shift, and is usually set to 0. An out-of-control signal occurs on the i^{th} sample when $S_i \geq h$ or or $S_i \leq -h$ for upward and downward mean shifts respectively.

2.2 The p-Value CUSUM Approach

In the context of hypothesis testing, early testing procedures make decisions using the concepts of rejection region and acceptance region. A null hypothesis would be rejected when the observed value of the related test statistic falls in the rejection region. This conventional way of hypothesis testing has been replaced by the p-value approach in recent literatures, because the p-value approach cannot only make a decision about the hypotheses, but also tell us how strong the evidence in the observed data is against the null hypothesis. Motivated by the p-value approach in hypothesis testing, suggestion was made designing control charts using the p-value approach as well. By the p-value approach, for a given control chart, the in-control (IC) distribution of the charting statistic is first computed or estimated. Then, at a given time point, the p-value corresponding to the observed value of the charting statistic can be obtained. If the p-value is less than a pre-specified significance level, then the chart signals a process distributional shift. Compared to conventional control charts using control limits, the p-value approach has several benefits. First, at a given time point, even if a shift is not detected, the p-value can provide a quantitative measure of the likelihood of a potential shift, so that the subsequent sampling interval can be adjusted properly. Second, conventional control charts may take different forms (e.g., the one-sided or two-sided charts) and their control limits are different in different situations. As a comparison, all control charts using the p-value approach have the same format, in the sense that the vertical axis is always in the range of (0, 1), denoting the p-values, and there is only one control limit corresponding to the significance level. This makes the charts more convenient to use. The p-value approach as a way of detecting mean shifts in Phase II Statistical Process Control (SPC) will be carried out in this research work.

2.3 Span Test

The p-value approach in designing Phase II CUSUM charts for detecting mean shifts of univariate processes will be described. The in-control (IC) process distribution is often assumed known in Phase II SPC. However, in practice, the IC process distribution is rarely known. Instead, it needs to be estimated from an IC dataset obtained at the end of Phase I analysis after the process has been adjusted properly so that it works stably. Hawkins and Olwell [8] proposed the self-starting CUSUM chart for Phase II SPC in cases when the IC process distribution is assumed normal but its mean and variance parameters need to be estimated.

2.4 The Design of p-Value Cumulative Sum (p-CUSUM)

Assume that $X_1, X_2, \dots X_t$ are a sequence of independent observations obtained during Phase II process monitoring. Their cumulative distribution functions (cdfs) are the same to be F_0 up to an unknown time point T and change to another cdf F_1 after the time point T. For simplicity, F_0 and F_1 is further assumed to be the same distribution except for their means μ_0 and μ_1. Then, the process has a mean shift at T, and the major goal of Phase II process monitoring is to detect the mean shift as soon as possible. To this end, the conventional CUSUM chart for detecting an upward mean shift uses the charting statistic:

$$\begin{cases} C_0^+ = 0, \\ C_0^+ = \max(0, C_t^+ + X_t - \mu_0 - k) \; for \;\; t \geq 1. \end{cases} \quad \textbf{(1)}$$

where k is an allowance constant.

2.5 Assessment of the Performance of p-Value CUSUM Chart

A number of authors have studied the design of CUSUM control scheme based on Average Run Length (ARL) computation. Instead of using ARL at a given quality level which is the average number of samples taken before an out of control signal is detected, the P-value is used. The chart gives a signal of mean shift when $C_t^+ > h$, where h is a control limit (Decision interval) chosen to achieve a pre-specified IC ARL (denoted as ARL0) value. Instead of comparing the charting statistic C_t^+ with the control limit (Decision interval) value h; here, computing the p-value corresponding to the value of $C_t^+$and then comparing the p-value with a pre-specified significance level α for making a decision whether the process is out-of-control (OC) are used. Hence, the need to find the IC distribution of C_t^+first. Recently, some researchers provided an approximation formula for this IC distribution in cases when the IC process distribution is normal with a known variance [10, 11]. The derivation of this formula is part theoretical and part empirical.

Assume that the IC process distribution is completely known. Then, for a given time point $t \geq 1$ and a given allowance constant k, we generate Phase II observations $X_1, X_{2,} \dots X_T$ and compute the value of $C_t^+$by (1). This process is then repeated many times (e.g., 1 million times), and the empirical distribution of C_t^+can be determined by the computed C_t^+values. For a given observed value of C_t^+, denoted as C_t^+, the corresponding p-value is then computed by:

$$p_t^+ = P(C_t^+ > C_t^{+*}) \quad \textbf{(2)}$$

Where C_t^{+*} are the CUSUM values greater than zero (0).

The IC process distribution is the normalized versions of the standard normal N(0, 1), it can be seen that the p-values, which are the right-tail probabilities of the IC distribution of C_t^+ depend on t; but they are stable when $t \geq 50$, which is consistent with the well-known steady-state distribution of C_t^+ [8]. Therefore, to use the p-values to monitor the process, there is need to compute the probability distributions of $C_t^+$for t < 50. The p-values depend slightly on the IC process distribution. For several commonly used significance levels α=0.01, 0.02, 0.05, and 0.10, the corresponding critical values (CVs) of C_t^+ are provided, in cases when k = 0.25 or 0.5, and the IC process distribution is the normalized version of N(0, 1). The corresponding ARL0 values are also provided. These values are all computed based on 1 million simulation runs for C_t^+.

2.6 Choosing the p-Values for the p-Value CUSUM Chart

Fig. 1 presents the p-values computed by equation (2) in cases when the IC distribution of C_t^+ is determined by 1 million replications, k = 0.5, t = 1, 5, 10, 50 and 100, and the IC process distribution is the normalized versions of the standard normal N (0, 1)

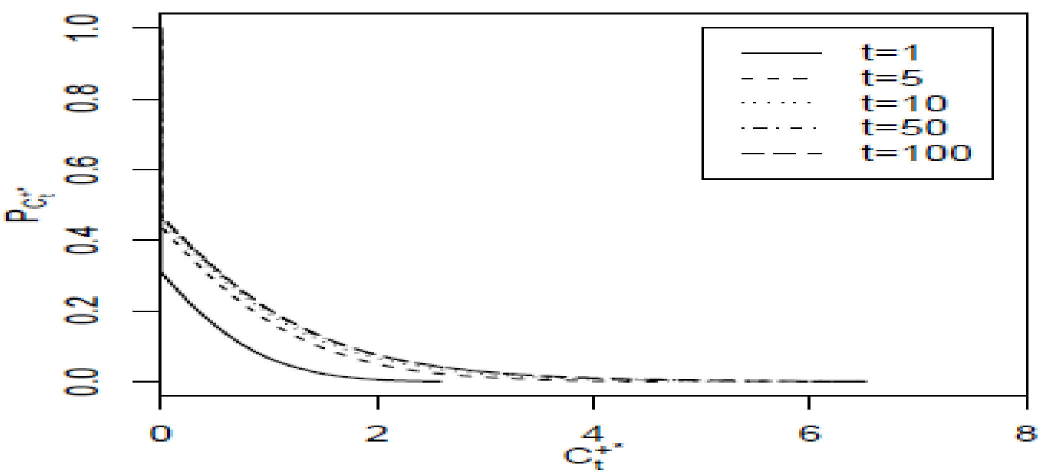

Fig. 1: Monograph of p-values computed using equation (2) with different t and IC process distributions

From Fig. 1, it can be seen that the p-values, which are the right-tail probabilities of the IC distribution of C_t^+ depend on t, but they are stable when t ≥ 50, which is consistent with the well-known steady-state distribution of C_t^+ [8]. Therefore, in practice, to use the p-values to monitor the process, the need to compute the probability distributions of C_t^+, for t < 50. From Fig. 1, it can also see that the p-values depend slightly on the IC process distribution.

For several commonly used significance levels α = 0.01, 0.02, 0.05, and 0.10, the corresponding critical values (CVs) of C_t^+ are provided in Table 1, for cases when k = 0.25 or 0.5, and the IC process distribution is the normalized version of N (0, 1).

Table 1: Critical values (CVs) and the corresponding *ARL₀* values for several commonly used significance level α = 0.01, 0.02, 0.05 and 0.10 and several IC process distributions

		α=0.01		α=0.02		α=0.05		α=0.1	
K		**CV**	ARL_0	**CV**	ARL_0	**CV**	ARL_0	**CV**	ARL_0
N(0,1)	**0.25**	**8.1841**	**996.238**	**6.1967**	**464.853**	**5.2237**	**172.234**	**3.9236**	**71.278**
	0.50	**4.0606**	**322.823**	**3.3483**	**169.538**	**2.4170**	**56.003**	**1.7237**	**25.425**
t_4	**0.25**	**8.8185**	**1262.625**	**7.2411**	**576.815**	**5.2305**	**195.717**	**3.7918**	**82.608**
	0.50	**4.9217**	**506.946**	**3.7781**	**218.192**	**2.5281**	**73.0202**	**1.6415**	**33.620**
χ_1^2	**0.25**	**11.5085**	**1153.281**	**9.5924**	**582.298**	**6.1924**	**183.958**	**5.0404**	**86.091**
	0.50	**7.3315**	**481.008**	**5.8988**	**233.748**	**4.0530**	**82.886**	**2.6607**	**35.554**
χ_4^2	**0.25**	**9.9038**	**1048.514**	**8.3649**	**512.733**	**6.1924**	**183.958**	**4.5247**	**76.111**
	0.50	**5.6788**	**419.965**	**4.6678**	**203.735**	**3.3290**	**68.505**	**2.2905**	**28.742**

From Table 1, it can be seen that the critical values (CVs) and the ARL0 values decrease when k increases or C_t^+ increases. To use the CVs in Table 1, one can have a rough idea about the p-value after the value of the charting statistic is computed. For instance, when the IC process is N (0, 1) and k = 0.25, if the computed value of the charting statistic is 8.2, then from Table 1, it is observed that the corresponding p-value would be less than 0.01 because 8.2 is larger than 8.1841.

2.7 p-Value Cumulative Sum Chart Procedures

To design CUSUM chart using p-values, there is need to choose the parameters (k, α and d) properly such that the pre-specified ARL_0 value is achieved. Firstly, a reference value k which is half of a target mean shift was selected. The resulting control chart is expected to be good for detecting small mean shifts if chosen k is small, and it will equally be good for detecting large mean shifts if chosen k is large. Secondly, the sampling intervals d was properly determined. Usually, $d_1$is chosen to be the shortest time to sample an item, and d_2 is chosen to be $2 - d_1$. In such cases, the average of d_1 and d_2 equals d_1 = 1. Lastly, if the value of ARL_0 was given before hand, then α was chosen such that the CUSUM chart using p-values and a fixed sampling scheme achieves the pre-specified ARL_0 value. It is also possible to specify the value of α in advance (e.g., α = 0.05), as in the hypothesis testing set up. Then, the value of ARL_0 of the CUSUM chart using p-values and a fixed sampling scheme can be determined accordingly. A statistical software (R) was used to analyzed the data and plot the charts in this study.

3.0 Results and Data Analyses

3.1 Ordinary CUSUM Control Chart

Fig. 2 depicts the CUSUM chart on Inflation rates in Nigeria. The CUSUM statistic: $S_i=\sum_{i=1}^{108} x_i - k$ are obtained and plotted against the sample number using their respective standard CUSUM parameters, reference value "k" (1.5) and decision interval "h" (5) using a statistical software (Anygeth.EXE) to obtain the CUSUM chart in Fig. 2.

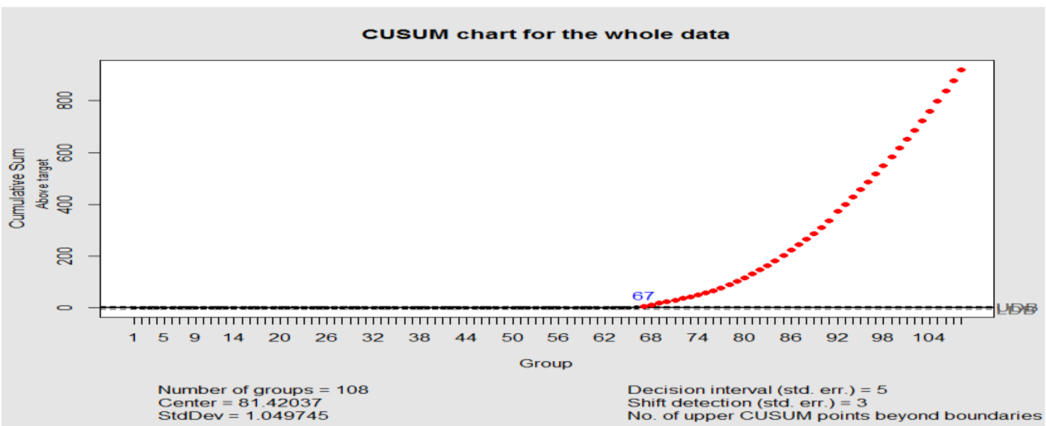

Fig. 2: CUSUM chart on the whole inflation rates (2002-2011)

The chart shows the CUSUM chart on the whole inflation rates data set. The CUSUM for the whole data shows that the process appears to be well within its allowable range in the early part of the data set. The upper CUSUM is the focus because only increase in inflation needs critical attention. Based on this, the first 66 observations are used as an IC data, and the remaining observations are used for testing. From the CUSUM chart in Fig. 2, shift was detected at sample point 67 which corresponds to July, 2008.

3.2 p-Value CUSUM Control Chart

The process mean always appears to be well within its allowable range in the early part of the data sets [9]. Based on this result, the first 66 observations are used as an IC data, and the remaining observations are used for testing. In this study, the standard k =1.5, and α= 0.05 were set. The IC data has a mean of 65.8576 and standard deviation of 11.1473. Next, the IC data is standardized to have mean 0 and variance 1. Using the standardized data, the procedure proposed by Spiegelhalter *et al* [12] was applied to obtain the p values as presented in Table 2. The p-CUSUM Chart in Fig.3 is a plot of the p-values in Table 2 against their corresponding months. The plot showed the p-value CUSUM for the inflation rates. The p-value CUSUM Chart signals a shift at the 65th time point which corresponds to May, 2008.

Table 2: Values of CUSUM and their respective p-value

S/No.	CUSUM values	p-values
1	0	1
2	0	1
3	0	1
4	0	1
5	0	1
6	0	1
7	0	1
8	0	1
9	0	1
10	0	1
11	0	1
12	0	1
13	0	1
14	0	1
15	0	1
16	0	1
17	0	1
18	0	1
19	0	1
20	0	1
21	0	1
22	0	1
23	0	1
24	0	1
25	0	1
26	0	1
27	0	1
28	0	1
29	0	1
30	0	1
31	0	1
32	0	1
33	0	1
34	0	1
35	0	1
36	0	1
37	0	1
38	0	1
39	0	1
40	0	1
41	0	1
42	0	1
43	0	1
44	0	1
45	0	1
46	0	1
47	0	1
48	0	1
49	0	1
50	0	1
51	0	1
52	0	1
53	0	1
54	0	1
55	0	1
56	0	1
57	0	1
58	0	1
59	0	1
60	0	1
61	0	1
62	0	1
63	0	1
64	0	1
65	0.032688	3.22E-21
66	0.872746	5.58E-21
67	2.224139	1.49E-20
68	3.844654	5.87E-20
69	5.734293	0.000621
70	7.462458	0.00011
71	9.190622	1.96E-05
72	11.10717	2.88E-06
73	13.15829	3.71E-07
74	15.37087	4.06E-08
75	17.74493	3.78E-09
76	20.25356	3.07E-10
77	23.21969	1.58E-11
78	26.64333	5.16E-13
79	30.60522	9.82E-15
80	34.86314	1.39E-16
81	39.25563	1.72E-18
82	43.78267	1.86E-20
83	48.47119	1.71E-22
84	53.75179	8.70E-25
85	59.27459	3.48E-27
86	65.30873	8.33E-30
87	71.31595	2.05E-32
88	77.53847	4.07E-35
89	83.761	8.07E-38
90	90.8178	6.95E-41
91	98.17064	4.45E-44
92	108.2147	1.93E-48
93	116.2404	6.33E-52
94	124.3468	1.91E-55
95	132.4801	5.60E-59
96	140.9901	1.13E-62
97	149.877	1.56E-66
98	159.0598	1.60E-70
99	168.6733	1.07E-74
100	178.1253	8.41E-79
101	187.8465	5.05E-83
102	197.8905	2.19E-87
103	208.0422	8.55E-92
104	218.7322	1.95E-96
105	229.8797	2.81E-101
106	241.1886	3.44E-106
107	252.5245	4.11E-111
108	264.2102	3.46E-116

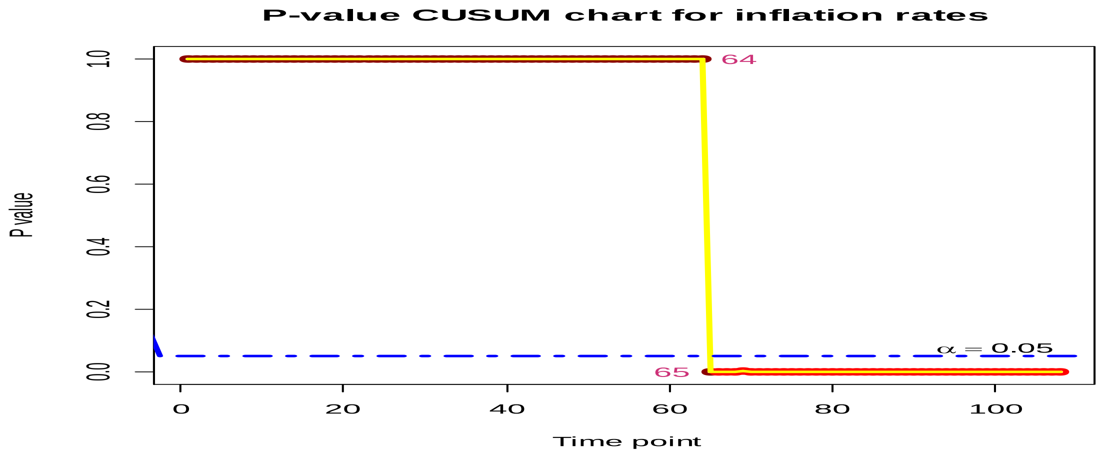

Fig. 3: p-CUSUM Chart using R Software

4.0 Discussion

From the analysis presented in Table 2 and p-value control chart (Fig. 3), it can be observed from the local means estimates that the rate of inflation increases per month from the 67th to 108th segments (i.e. from May 2008 to December 2011). Using Anygeth-Statistical software, the standardized value of the reference value denoted as "k" was given as 1.5 and the decision interval denoted as "h" was given as 5, which were applied to the inflation rates data. From the CUSUM and the corresponding p-values for the period under study as presented in Table 2, the values in columns 2 and 3 were plotted against column 1 (time period) as depicted in Figs. 2 and 3 (CUSUM and p-CUSUM charts). It was observed that the first out of control signal was at sample point 67 and 65 which corresponds to May and July, 2008 respectively. Since the shift in the process mean was noticed earlier in p-CUSUM than ordinary CUSUM chart, it is an evidence that p-CUSUM is a preferred control tool to be used than the ordinary CUSUM. This is applicable when the control experimenter or statistician desires to start controlling a process at an earlier stage [12].

5.0 Conclusion and Recommendation

On the basis of our analyses, it is evident that p-value CUSUM chart scheme is more effective than ordinary CUSUM at detecting small shift in the process mean in this type of data set. Hence, it is recommended that the Central Bank of Nigeria (CBN) and other bodies in charge of monitoring inflation rates can preferably adopt the use this method of statistical process control in monitoring rates of inflation.

Acknowledgement

Authors are grateful to the management of Central Bank of Nigeria (CBN), Ilorin, Kwara State Branch for providing the data and other relevant information that has made this study successful.

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