Effect of Sampling Errors on Single Sampling Plan Rectifying Inspection

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Abstract

Quality control systems can be divided into process control and product control. Inspection plans play a vital role in product control and must be used wisely. Recently, researchers in various quality control procedures consider the possibility of inspection errors as an important issue. To assess the performance of Single Sampling Plan, Average Outgoing Quality *(AOQ) and* , Average Total Inspection *(ATI) are used. The impacts of inspection error for varying producer's and consumer's risk are examined using AOQ and ATI to assess the performance. A statistical software R (R-package) was used to obtain the two parameters and their plots at varying producer's and consumer's risk points. From the analyses, increase in both producer's and consumer's risks (α, β) resulted in reduced AOQ for* $p \leq 0.06$. *Between* $p = 0.07$ *and* $p = 0.08$, *there was no pattern. However, when* $p \geq 0.09$, *as α and β increase, AOQ also increases.*

Keywords: Average Outgoing Quality, Average Total Inspection, Producer's Risk, Consumer's Risk, Probability of Acceptance

Abbreviations and Acronyms

P – Fraction of items defective
P_e – Apparent fraction defective
e_1 (α) – Type I inspection error (Producer's Risk)
e_2 (β) – Type II inspection error (Consumer's Risk)
N – Lot size
n – Sample size
X – Value for measurable quality characteristic in variable sampling plans for fraction defective
x – Sample
P_a – Probability of acceptance for a single variable sampling plan for fraction defective.
OC – Operating Characteristic
C – Acceptance number
D – Observed number of defectives
AOQ – Average outgoing quality
ATI – Average Total Inspection
LTPD – Lot Tolerance Percent Defective
AQL – Acceptable Quality Level

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1.0 Introduction

Quality control has become one of the most important tools that distinguish different commodities in a global business market. Two important techniques for ensuring quality are the statistical product control in the form of acceptance sampling and statistical process control. Acceptance sampling is an important field of statistical quality control used to accept or reject products slated for inspection. This field was popularized by Balakrishnan *et al*. [1] where the procedure of Acceptance Sampling was summarized as a sample randomly taken from a lot and the fate of the products depends on the information obtained from this sample. Therefore, acceptance sampling is used for possible acceptance or rejection of the products but not for estimating the quality of the lot.

Acceptance sampling plan is a `middle path' between hundred percent inspections and no inspection at all. Products may be grouped into lots or may be single pieces from a continuous operation. A sample is selected and checked for various characteristics. For products grouped into lots, the entire lot is either accepted or rejected. The decision is based on the specified criteria and the amount of defects or defective units found in the sample. Sampling at the end of manufacturing process provides a check on the adequacy of the quality control procedure of the manufacturing department. The product is accepted, that is passed on to the next organization or customer. The sampling procedures will prevent defective products from going any further. The manufacturing department, as part of the process or quality control program, may also be sampling techniques, [2–4].

Some firms initiate total quality management (TQM) programs to ensure high levels of quality. This emphasizes that no defect should be passed from producers to customers, whether the consumer is external or internal. However, in reality, many firms still rely on checking their materials inputs. Consumers need acceptance sampling to limit the risk of rejecting good quality material or accepting bad quality materials. Consequently, the consumer specifies the parameters of the plan. Company can be both a producer of goods purchased by another company and a consumer of goods or raw materials supplied by another company. Acceptance sampling plan precisely specifies the parameters of the sampling process and the acceptance/rejection criteria. The variables to be specified include the lot size (N), the size of the sample to be inspected from the lot (n) and the number of defects above which a lot is rejected (c) in (d). Acceptance sampling plan could be single, double, multiple or chain [5].

The present study is aimed at studying the effect of inspection errors on single acceptance sampling plan rectifying inspection and the impact of inspection errors (type I (α) and type II (β)) on this plan with the sole objective of determining the effect of varying these error values on Average Outgoing Quality (AOQ) and Average Total Inspection (ATI).The effect of inspection error on this plan will be evaluated through operating characteristics curves.

2.0 Materials and Methods

Two types of errors are possible in attribute sampling. An item which is good may be classified as defective (type I error, e_1), or an item that is defective may be classified as good (type II error, e_2). Hence, for attribute sampling, the apparent fraction of defective items in a lot is defined as:

$$p_e = p(1 - e_2) + (1 - p)e_1 \quad (1)$$

where p represents the true fraction of defective items in the lot.

2.1 Sampling Plans and the Effects of Inspection Errors

If N and p represent lot size and true fraction of defective items in the lot, the AOQ of the inspection with replacement can be written as:

$$AOQ = \frac{npe_2 + p(N-n)(1-p_e)e_1 + p(N-n)(1-p_{ae})e_2}{N - np_e + p(N-n)(1-p_{ae})(S-n)p_e} \quad (2)$$

Where n = sample size, e_1 = Probability of type I error, e_2 = Probability of type II error

p_e = Apparent fraction of defective items, Pa_e = Probability of acceptance with inspection error, given by

$$p_{ae} = \sum_{x=0}^{c} \binom{n}{x} {p_e}^x (1 - p_e)^{n-x} \quad (3)$$

Similarly, the expression for ATI for the inspection process without replacement is

$$ATI = \frac{n+(1-p_{ae})(s-n)}{1-p_e} \tag{4}$$

2.2 Mathematical Development

The following form of the marginal distribution of x has been derived by some researchers [6–8].

$$g_n(x) = \binom{n}{x} P_x (1-P)^{n-x}, \;\; x = 0, 1, . . ., n \tag{5}$$

Under the assumption that the number of defectives X in a lot size N is binomially distributed, with a p.d.f:

$$f_N(X) = \binom{N}{X} P_x (1-P)^{N-X} \qquad x = 0,1 , . . ., N \tag{6}$$

where p is the process fraction defective.

The second assumption of equation (5) is that the number of defectives x in a sample size n given X is hypergeometric:

$$f(x/X) = \frac{\binom{n}{x}\binom{N-n}{X-x}}{\binom{N}{X}} \tag{7}$$

Thus, this proves that the Hald's derivation of the binomial distribution was reproduced by hypergeometric sampling. Thus for the Bayesian operating characteristics (BOC) curve, the probability of lot acceptance is derived from the above equations as:

$$P_a = \sum_{x=0}^{c} g_n(x) = \sum_{x=0}^{c} \binom{n}{x} P_x (1-P)^{n-x}, \tag{8}$$

$x = 0, 1, . . ., n \;\; 0 \le p \le 1$

where c is the acceptance number.

For the inspection error analysis, the observed defectives from a sample is replaced by observed number of defectives y_e. The probability of lot acceptance given in (8) will be reduced:

$$P_{ae} = \sum_{y_e=0}^{c} g_n(y_e) \tag{9}$$

Where:

$$g_n(y_e) = \binom{n}{y_e} {P_e}^{y_e} (1-P_e)^{n-y_e}, \;\; y_e = 0, 1, \ldots, n, \tag{10}$$

Equation (8) gives the probability of lot acceptance for perfect inspection. The probability of lot acceptance when inspection errors are presented as P_{ae} in equation (9), and using equation (10) the expression derived becomes:

$$P_{ae} = \sum_{y_e=0}^{c} \binom{n}{y_e} {P_e}^{y_e} (1-P_e)^{n-y_e} \tag{11}$$

The expression for average outgoing quality (AOQ) is:

$$AOQ = \frac{\text{expected number of defectives items remaining after inspection}}{\text{Total number of items in the lot}} \tag{12}$$

$$= \frac{(N-n)pP_a}{N}$$

An expression from AOQ can be derived by introducing the following terms: p(N - M), the number of defectives in the uninspected portion of an accepted lot p(N - M)e_2, the number of defectives items classified as being good in the screened portion of the rejected lot; $n_p e_2$ is the number of defective items classified as good in the sample; DITR is the number of defective items introduced through replacement into the lot. For an accepted lot, the expected number of defective items replaced in the lot is:

$$y = np_e \tag{13}$$

The probability that an item is classified as being good is then:

$$P_g = (1 - P)(1 - e_1) + pe_2 \tag{14}$$

A set of n_1 items is selected at random, tested and classified as good or bad. A total of np_e items were needed to replace the defective items in the accepted lot. This procedure of sampling is defined as negative binomial process. The expected number of items tested to obtain np_e items, which are good, is then:

$$\frac{y}{p_g} \tag{15}$$

The expected number of defective items replaced in an accepted lot is then:

$$DITR_a = Pe_2\left(\frac{y}{P_g}\right) \tag{16}$$

The expected number of defective items replaced in a rejected lot, which was screened, is:

$$DITR_s = pe_2 \frac{(N-n)P_e}{P_g} \tag{17}$$

The expected number of items to be replaced is:

$$DITR = Pe_2\left(\frac{y}{P_g}\right) + pe_2 \frac{(N-n)P_e}{P_g}(1 - P_{ae}) \tag{18}$$

$$= \frac{Pe_2}{P_g}[y + (N-n)P_e(1 - P_{ae})] \tag{19}$$

The expression AOQ is then:

$$AOQ = \frac{p(N-n)P_{ae} + p(N-n)(1-P_{ae})e_2 + npe_2 + DITR}{N} \tag{20}$$

Expression (18) can be reduced to the form:

$$AOQ = \frac{nPe_2 + p(N-n)(1-P_e)P_{ae} + p(N-n)(1-P_{ae})e_2}{N(1-P_e)} \tag{21}$$

Similarly the AOQ expression for sampling with no replacement can be derived as:

$$AOQ = \frac{p(N-n)P_{ae} + p(N-n)(1-P_{ae})e_2 + npe_2}{N - np_e - (1-P_{ae})(N-n)P_e} \tag{22}$$

This is true; hence no defectives are introduced through the replacement process.

The Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) are computed at varying producer's and consumer's risk points for Single Sampling Plan (SSP) with sample size n=100, acceptance number c = 4 and quality level p = 0.1 to 0.5 using R language Statistical Software (R version 2.15.3) [9]. The effect of varying each of the producer's and consumer's risk on apparent AOQ and ATI respectively is examined in this study.

3.0 Results and Analyses

3.1 Effect of increase in Producer's and Consumer's Risk on AOQ and ATI

Table 1 and Fig. 1 show the effect of increase in both producer's and consumer's risk with reference to AOQ. The information indicate that as (α, β) combinations increase, the values of AOQ decrease for P ≤ 0.06. Between P = 0.07 and P = 0.08, there is no pattern while beyond P ≥ 0.09, as (α, β) increase the AOQ increase. Table 2 and Fig. 2 present the effect of increase in producer's and consumer's risk with respect to ATI. The data revealed that ATI uniformly increase as (α, β) increase.

Table 1: Effect of increase (α, β) on AOQ

	(0.05,0.05)	(0.04,0.04)	(0.03,0.03)	(0.02,0.02)	(0.01,0.01)	(0.0,0.0)
P	AOQ1	AOQ2	AOQ3	AOQ4	AOQ5	AOQ6
0.01	0.0010	0.0042	0.0058	0.0073	0.0084	0.0025
0.02	0.0041	0.0059	0.0084	0.0115	0.0146	0.0031
0.03	0.0044	0.0060	0.0087	0.0125	0.0170	0.0027
0.04	0.0043	0.0055	0.0077	0.0112	0.0160	0.0019
0.05	0.0042	0.0048	0.0063	0.0090	0.0133	0.0013
0.06	0.0042	0.0044	0.0051	0.0068	0.0098	0.0008
0.07	0.0045	0.0042	0.0043	0.0050	0.0068	0.0004
0.08	0.0049	0.0042	0.0038	0.0039	0.0046	0.0002
0.09	0.0053	0.0044	0.0037	0.0032	0.0031	0.0001
0.1	0.0059	0.0048	0.0038	0.0029	0.0022	5.93e-05
0.15	0.0092	0.0073	0.0054	0.0036	0.0018	1.06e-06
0.20	0.0130	0.0103	0.0077	0.0051	0.0025	9.69e-09

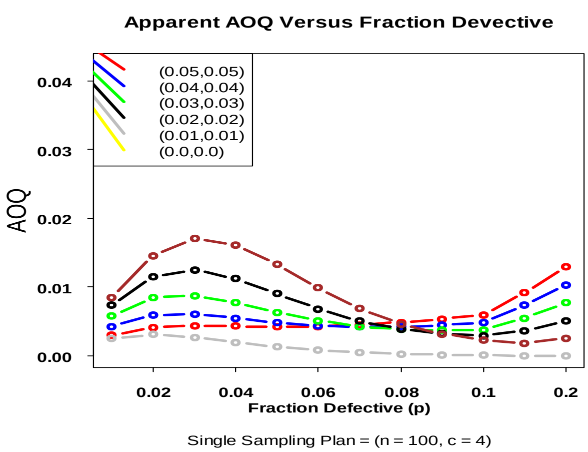

Figure 1: Apparent AOQ versus Fraction Defective

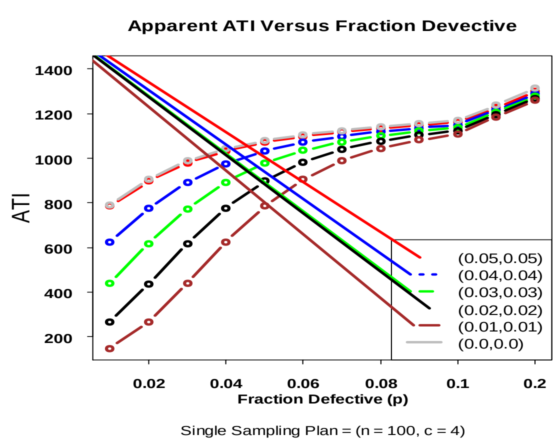

Figure 2: Apparent ATI versus Fraction Defective

Table 2: Effect of increase in (α, β) on ATI

	(0.05,0.05)	(0.04,0.04)	(0.03,0.03)	(0.02,0.02)	(0.01,0.01)	(0.0,0.0)
P	ATI1	ATI2	ATI3	ATI4	ATI5	ATI6
0.01	784.6941	625.3194	440.6209	265.7623	147.0855	791.8156
0.02	896.8965	776.0107	618.1118	436.7989	265.7623	907.3253
0.03	977.6081	892.6152	773.0829	618.1118	440.6209	988.4258
0.04	1033.328	976.114	892.6152	776.0107	625.3194	1043.076
0.05	1071.452	1033.328	977.6081	896.8965	784.6941	1079.803
0.06	1098.231	1072.151	1035.344	982.0167	905.2706	1105.469
0.07	1118.204	1099.236	1074.213	1039.275	989.1232	1124.817
0.08	1134.354	1119.374	1101.208	1077.546	1044.928	1140.815
0.09	1148.51	1135.674	1121.289	1104.075	1082.002	1155.192
0.1	1161.729	1150.012	1137.626	1123.902	1107.739	1168.897
0.15	1226.976	1216.51	1206.205	1196.043	1185.996	1238.381
0.20	1298.701	1288.659	1278.772	1269.034	1259.444	1315.789

3.2 Effect of varying Consumer's Risk and fixing Producer's Risk on AOQ and ATI

Table 3 and Fig. 3 show the effect of varying consumer's risk and fixing producer's risk. The data indicated that as the AOQ increases, the consumer's risk increase when other factors are fixed. The effects of varying consumer's risk and fixing producer's risk are presented in Table 4 and Fig. 4. The data indicated that the ATI decrease as the consumer's risk decrease when others factors are fixed.

Table 3: Effect of varying consumer's risk on AOQ

	(0.05,0.04)	(0.05,0.03)	(0.05,0.02)	(0.05,0.01)	(0.05,0.00)
P	AOQ1	AOQ2	AOQ3	AOQ4	AOQ5
0.01	0.0029	0.0028	0.0027	0.0026	0.0025
0.02	0.0039	0.0037	0.0035	0.0033	0.0031
0.03	0.0040	0.0037	0.0033	0.0030	0.0027
0.04	0.0038	0.0033	0.0029	0.0024	0.0019
0.05	0.0036	0.0030	0.0024	0.0019	0.0013
0.06	0.0036	0.0028	0.0022	0.0015	0.0008
0.07	0.0037	0.0029	0.0021	0.0012	0.0004
0.08	0.0039	0.0030	0.0021	0.0012	0.0002
0.09	0.0043	0.0033	0.0022	0.0012	0.0001
0.1	0.0047	0.0036	0.0024	0.0012	5.93e-05
0.15	0.0074	0.0055	0.0037	0.0019	1.06e-06
0.20	0.0104	0.0078	0.0052	0.0026	9.69e-09

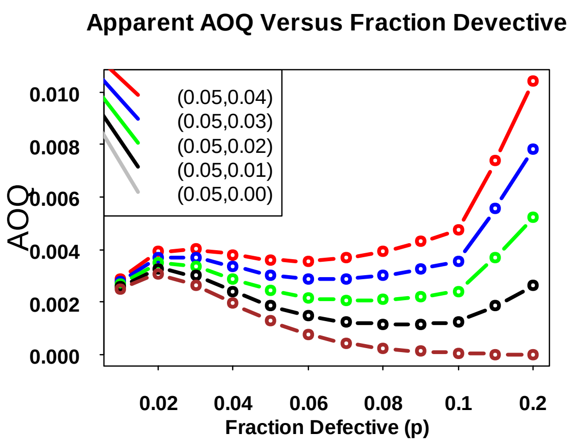

Fig. 3: Apparent AOQ versus Fraction Defective

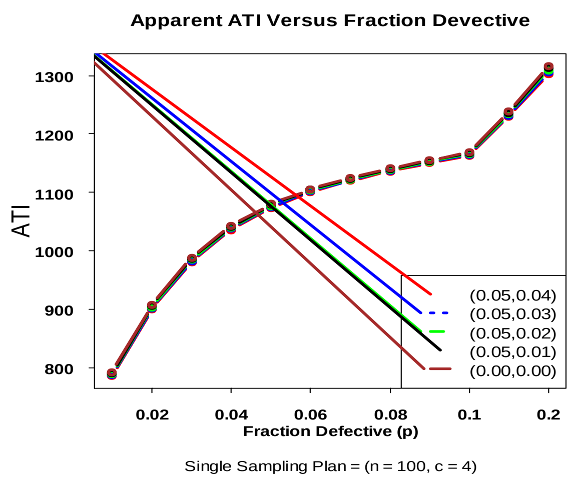

Figure 4: Apparent ATI versus Fraction Defective

Table 4: Effect of varying Consumer's risk on ATI

	(0.05,0.04)	**(0.05,0.03)**	**(0.05,0.02**	**(0.05,0.01)**	**(0.05,0.00)**
P	**ATI 1**	**ATI 2**	**ATI 3**	**ATI 4**	**ATI 5**
0.01	786.1268	787.5553	788.9796	790.3997	791.8156
0.02	899.0135	901.1149	903.2005	905.2706	907.3253
0.03	979.8262	982.0167	984.1799	986.3162	988.4258
0.04	1035.344	1037.326	1039.275	1041.191	1043.076
0.05	1073.188	1074.89	1076.559	1078.197	1079.803
0.06	1099.734	1101.208	1102.655	1104.075	1105.469
0.07	1119.567	1120.909	1122.231	1123.533	1124.817
0.08	1135.674	1136.978	1138.27	1139.549	1140.815
0.09	1149.862	1151.205	1152.541	1153.87	1155.192
0.1	1163.169	1164.605	1166.038	1167.468	1168.897
0.15	1229.241	1231.514	1233.795	1236.084	1238.381
0.20	1302.083	1305.483	1308.9	1312.336	1315.789

3.3 Effect of varying Producer's Risk and fixing Consumer's Risk on AOQ and ATI

Table 5 and Fig. 5 show the effect of varying producer's risk and fixing consumer's risk. The data indicated that as producer's risk increase the AOQ increase. Table 6 and Fig. 6 indicate the effect of varying producer's risk and fixing consumer's risk. As (α, β) increases, the ATI decreases.

Table 5: Effect of varying producers risk on AOQ

	(0.04,0.03)	(0.05,0.03)	(0.02,0.03)	(0.01,0.03)	(0.00,0.03)
P	AOQ1	AOQ2	AOQ3	AOQ4	AOQ5
0.01	0.0041	0.0041	0.0040	0.0040	0.0050
0.02	0.0057	0.0055	0.0053	0.0051	0.008
0.03	0.0057	0.0053	0.0050	0.0047	0.0097
0.04	0.0050	0.0045	0.0040	0.0036	0.0107
0.05	0.0042	0.0036	0.0030	0.0024	0.0114
0.06	0.0036	0.0029	0.0022	0.0015	0.0122
0.07	0.0033	0.0025	0.0017	0.0009	0.0132
0.08	0.0033	0.0023	0.0014	0.0005	0.0144
0.09	0.0034	0.0023	0.0012	0.0003	0.0158
0.1	0.0036	0.0025	0.0012	0.0001	0.0174
0.15	0.0055	0.0037	0.0018	2.44e-06	0.0268
0.20	0.0078	0.0052	0.0026	2.33e-08	0.0376

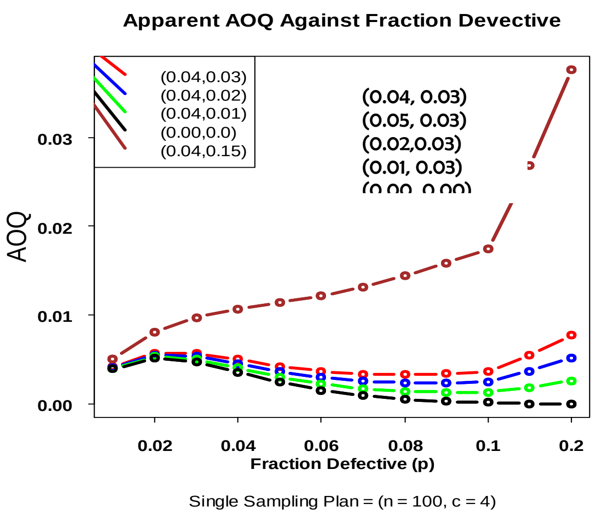

Figure 5: Apparent AOQ against Fraction Defective

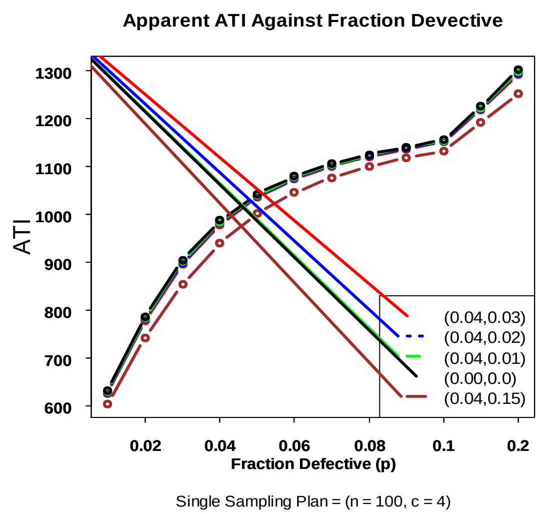

Figure 6: Apparent ATI against Fraction Defective

Table 6: Effect of varying producers risk on ATI

	(0.04,0.03)	(0.05,0.03)	(0.02,0.03)	(0.01,0.03)	(0.00,0.00)
P	ATI1	ATI2	ATI3	ATI4	ATI5
0.01	627.1143	628.9063	630.6954	632.4816	605.3939
0.02	778.9218	781.8163	784.6941	787.5553	742.9015
0.03	895.8321	899.0135	902.1596	905.2706	854.8363
0.04	979.0899	982.0167	984.895	987.7256	939.9457
0.05	1035.843	1038.305	1040.715	1043.076	1001.829
0.06	1074.213	1076.228	1078.197	1080.121	1045.843
0.07	1100.964	1102.655	1104.309	1105.929	1077.218
0.08	1120.909	1122.418	1123.902	1125.362	1100.229
0.09	1137.141	1138.591	1140.025	1141.445	1118.007
0.1	1151.503	1152.985	1154.458	1155.924	1132.684
0.15	1218.738	1220.975	1223.219	1225.47	1192.443
0.20	1291.989	1295.337	1298.701	1302.083	1253.129

4.0 Discussion

The data presented in the study indicated that inspection error has a strong impact on the performance of inspections and could be misleading if the trend of effect of error is not properly studied. From the results and analyses, the effects of inspection error on AOQ and AIT at various combinations were observed.

An increase in producer's and consumer's risk resulted in increase in AOQ for $p > 0.06$, while no particular pattern was observed between $p = 0.07\ and\ p = 0.08$. However, between $p > 0.09$, as (α, β) increased, AOQ also increased. Furthermore, increase in both producer's and consumer's risk (α, β) resulted in uniform increase in ATI. When consumer's risk (α) is varied while producer's risk (β) and fraction defective (P) were kept constant, AOQ increased as the consumer's risk increased. When the consumer's risk (β) is varied while producer's risk (α) and fraction defective (P) is kept constant, ATI decreased as the consumer's risk decreased. However, when the producer's risk is varied while consumer's risk and fraction defective were kept constant, the producer's risk increased as AOQ increased.

The implication of this study is that when an experimenter is designing a single acceptance sampling inspection, the values of α and β should not be set below 0.05 if the researcher desires a higher AOQ that will protect both the producer and the consumer.

5.0 Conclusion and Recommendation

In conclusion, in order to mitigate the effect of sampling errors, the present study recommends estimation of the inspection error for the inspections by using well-designed experiments. In the event that the level of errors is high, inspectors should be trained to minimize the errors.

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