Effect of Chemical Reaction on Mass Transfer Embedded in a Porous Medium over an Exponentially Porous Stretching Surface

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Abstract

The effect of chemical reaction on mass transfer embedded in porous medium is implemented over an exponentially porous stretching surface. The emerging governing equations were simplified using similarity variable function and some relevant dimensionless parameters. The modelled equations were solved using Runge-Kutta method with shooting techniques. This was done by converting the modelled equations to an initial value problem and solved using Maple 18. The numerical computations were presented in tabular and graphical forms for variable fluid parameters controlling the flow, chemical reaction and mass transfer. The results in form of velocity and concentration profiles, Skin friction and Sherwood number increased with varied parameters were discussed. The findings were compared with existing solutions and found to have minimal errors.

Keywords: Boundary flow, Runge-Kutta, Shooting Technique, Sherwood Number

1.0 Introduction

The effect of chemical reactions on mass transfer has been subject of discussion for several years, partly due to the importance of such reactions in fluid dynamics. A lot of problems have been generated in dynamics which are expected to be solved analytically, numerically or by experimentations. While analytical solution deals with the exactness of the problem, numerical implementation deals with approximate solutions with or without minimal errors. While the experimental aspect seems to be the best of the three ways or methods of analysing solution to fluid, the cost of experimentation might pose problems, where such instrument for the experiments are not readily available.

The boundary layer flow and the chemical reactions in mass transfer embedded in porous media is currently attracting further research. Also, the chemical reactions on mass transfer over an exponentially stretching surface are gaining the attention of researchers. The reactions of researchers over this subject seems inconclusive because of the practicability in engineering problems such as paper production, plastic and metal sheets preparation [1, 2].

The boundary layer flow gained attention in previous studies where analytical solution of fluid flow over an exponentially stretching porous sheet was provided [3]. These researchers included heat flux in porous medium and numerical presentation by means of Homotopy Analysis Method (HAM). Other researchers proposed for analysis of steady free convection heat and mass transfer over a stretching sheet with chemical reaction in comparison with exact solution using shooting technique method with Range-Kutta method [4]. Studies have also been carried out using a fundamental analysis of boundary layer fluid flow in a channel with heat source. These researchers included Soret effects and Slip condition to clarify the behaviour of skin friction coefficient, Nusselt number and Sherwood numbers [5]. Unsteady MHD flow and heat transfer of nanofluid over a permeable shrinking sheet with thermal radiation and chemical reactions were presented by Srinivas and Kishan [6]. In their studies, they included the porous parameter and the reactions over an exponentially porous stretching surface. This implies that literature on nonlinear boundary layers and MHD layer problems in various situations have been presented and outcome of the studies analysed.

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Mass transfer over porous medium over stretching media with chemical reactions are very much important; hence the interest of researchers in mass transfer generation. In view of its numerous and wide-range applications to various fields of human endeavours such as film production process, artificial fibres, among others, research into this area of research would be very significant. In the light of this, Alharbi *et al.* [7] researched on chemical reaction of heat and mass transfer in MHD Visco-Elastic fluid flow through porous parameters (permeability) of the porous media. By extension, stagnation flow of couple stress nanofluid was analysed by other researchers and the effects were expressed over an exponentially stretching sheet through a porous medium [8]. It was discovered that the heat transfer rate at the surface decreased with increased values of the magnetic and radiations parameters. Ishak [9] in his studies presented MHD boundary layer flow over an exponentially sheet in the presence of radiation effect. In recent times, several other researchers have dealt with various effects on MHD boundary layer flow due to an exponentially stretching sheet [10-12].

In order to estimate the mass transfer rate in the medium, there is need to determine the distribution of the chemical reaction in concentration field. The field is determined by solving the concentration equation, which is a statement of concentration parameters which are transformed mass equations. The term mass transfer means the tendency of a component in a mixture to travel from a region of high concentration to a region of low concentration. There are two basic modes of mass transfer: Diffusion and Convection. There is a close similarity between heat transfer and mass transfer in terms of the transport rate equation and transport conversion equation. The convection mass transfer is analogous to convection heat transfer and occurs between a moving mixture fluid species and an exposed solid surface. In this treatment, heat aspect is not involved. Like heat transfer rates, the species mass flux can be determined from the mass conservation field by solving the species mass conservation equation which is a statement of conservation of mass species.

A number of analytical solutions exist when solving modelled problems in fluid dynamics but difficult to achieve in most cases. Where the exact solutions cannot be achieved, the need for approximate solution in form of numerical analysis come to play and this gives room for comparisons among the researchers. Since most studies focus on finding the exact solution, numerical and/or experimental presentation of flow of different fluid flow has been reported. A review of articles in this area up to date revealed that most analytical studies use different methods, which unfortunately limit the range of reliability of the results. Also, the analytical results can be used to check calculation of numerical methods. Experimental presentation is the best but where the required implement are not available and the research studies have to be presented, the need to explore the earlier methods suffice.

In view of the need to get clear picture of this study, the researcher involved the use of Runge-Kuntta Method with shooting technique. The governing equations of chemical reaction and mass transfer in fluids are essentially nonlinear differential equations. Therefore, the objective of this paper is to present the effects of chemical reactions on mass transfer over an exponentially porous stretching surface embedded in a porous medium.

2.0 Materials and Methods

The emerging equation is out of a slit at origin $(x = 0, y = 0)$ and moving with non-uniform velocity in the presence of thermal radiation. The equations governing the fluid flow and mass transfer as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{v}{k}u \tag{2}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - R(C - C_\infty) \tag{3}$$

Here *u* and *v* are the components of the velocity in the directions of *x* and *y* respectively. *R* is the fluid concentration parameter; *K* is the thermal conductivity of the fluid. The appropriate boundary conditions for the problems are given based on the study:

$$u = u_0 \exp\left(\frac{x}{l}\right), \quad v = -v_0 \exp\left(\frac{x}{l}\right), \quad C = C_\infty \quad y = 0 \tag{4}$$

$$u \to 0, \quad C \to C_\infty \text{ as } \quad y \to \infty \tag{5}$$

Where $u(x)$ is the stream-wise velocity and $-v(x)$ is the velocity of the suction of the fluid, (u, v)= Velocity components, (x, y) = Coordinates, C= Concentration at the fluid. C_w= Concentration at the Rate Surface, C_∞= Concentration outside the flow, D= Coefficient of mass diffusion, S_c = Schmidt

The following similarity variable relations for u, v can be introduced as follows:

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x},\ \phi(\eta) = \frac{C - C_\infty}{C_\varpi - C_\infty},\ \eta = \frac{y}{l}\sqrt{\frac{\mathrm{Re}}{2} e^{\frac{x}{2l}}},\quad C = C_\infty + (C_w - C_\infty)e^{\frac{2x}{l}}\phi(\eta) \tag{6}$$

$$\psi(x, y) = \sqrt{2R_e}\,\upsilon\, e^{x/2L} f(\eta) \tag{7}$$

Where $\psi(x, y)$ is the stream function and (6) are dimensionless parameters. Substituting equation (6) and (7) into equations (1)-(3), the following equations (8)-(9)

$$\frac{d^3}{d\eta^3} f(\eta) + f(\eta)\left(\frac{d^2}{d\eta^2} f(\eta)\right) - \left(\frac{d}{d\eta} f(\eta)\right) - K\frac{d}{d\eta} f(\eta) = 0 \tag{8}$$

$$\frac{d^2}{d\eta}\phi(\eta) + Scf(\eta)\left(\frac{d}{d\eta}\phi(\eta)\right) - ScR_c\phi = 0 \tag{9}$$

Using appropriate boundary condition (10)

$$f(\eta) = \frac{1}{2},\ f'(\eta) = 1,\ \phi(\eta) = 1, \eta = 0,\ f'(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0\ \ as\ \ \eta \to 0 \tag{10}$$

With relevant parameters, the equations (8) and (9) be solved by first converting them to initial value problems. Therefore, equations (11)-(13) were set to be

$$f' = z, z' = p, \phi' = q \tag{11}$$
$$p' = -kz + z^2 - fp \tag{12}$$
$$q' = -Sc(fp - R\phi) \tag{13}$$

To integrate equations (11) to (13) as an initial value problem, the values for $f''(0)$ and $\phi'(0)$ were required but no such values were given in the boundary. The suitable guess values $f''(0)$ and $\phi'(0)$ were chosen and then integration was carried out. The researcher compared the calculated values for $f'(0)$ and $\phi(0)$ at $\eta = 4$ with the given boundary condition $f'(10) = 0$ and $\phi(10)$ adjusted the estimated values, $f''(0)$ and $\phi'(0)$, to give a better approximation for the solution. The researcher performed the series of values for $f''(0)$ and $\phi'(0)$, then applied a fourth-order Runge–Kutta method with shooting techniques with step-size $h = 0.01$. The above procedure was repeated until the results up to the desired degree of accuracy 10^{-5}.

3.0 Results and Discussion

The values of Skin friction and Sherwood number for varied K, Sc and R are presented in Tables 1, 2 and 3 respectively. To validate the numerical method for obtaining Sc, and R in this study, the present result was compared with the existing problem considered by some researchers [1, 13 15]. The results for concentration gradient as compared are presented in Table 4. The numerical evaluation for the solutions of the problems expressed in equations (8) and (9) was performed and the results illustrated graphically (Figs. 1–3).

Table 1: Values of Skin friction and Sherwood number for varied K

K	**Sc.**	***R***	$f^{11}(0)$	$-\phi^{1}(0)$
1	**0.1**	**0.1**	**-1.4142**	**0.1631**
2	**0.1**	**0.1**	**-0.3064**	**0.1441**
4	**0.1**	**0.1**	**-0.0255**	**0.1257**
10	**0.1**	**0.1**	**-0.0044**	**0.1218**

Table 2: Values of Skin friction and Sherwood number for varied *Sc*

K	**Sc.**	***R***	$f^{11}(0)$	$-\phi^{1}(0)$
1	**0.5**	**0.1**	**-1.4142**	**0.1631**
1	**1**	**0.1**	**-0.3438**	**0.1489**
1	**2**	**0.1**	**-0.3064**	**0.1441**
1	**3**	**0.1**	**-0.0255**	**0.1256**

Table 3: Values of Skin friction and Sherwood number for varied *R*

K	**Sc.**	***R***	$f^{11}(0)$	$-\phi^{1}(0)$
1	**0.1**	**2**	**-1.4142**	**0.1631**
1	**0.1**	**4**	**-0.3438**	**0.1489**
1	**0.1**	**6**	**-0.3064**	**0.1441**
1	**0.1**	**8**	**-0.2554**	**0.1257**

Table 4: Comparison with the existing Studies

Sc	R_c	Present Study	Elbashbeshy et al. [1]	Andersson et al. [13]	Takhar et al. [14]	Uddin et al. [15]
0.1	0.1	0.0148901	0.149083	0.14900	0.15042	0.15057
1	0.1	0. 666764	0.668754	0.66900	0.67044	0.66873
1	1	1.173206	1.176401	1.17700	1.17761	1.17679
10	1		3.871327	3.88000	3.87469	3.87347
10	10		10.241185	10.25000	10.24283	10.24535

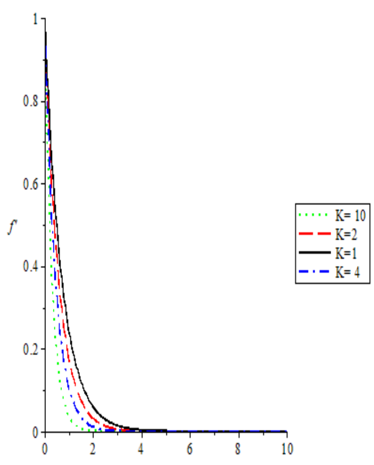

Figure1: Velocity profile K

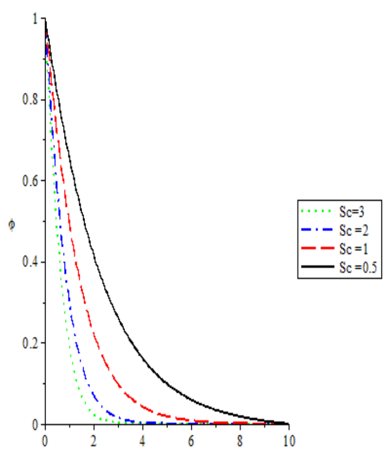

Figure 2: Concentration Profile for varied Sc

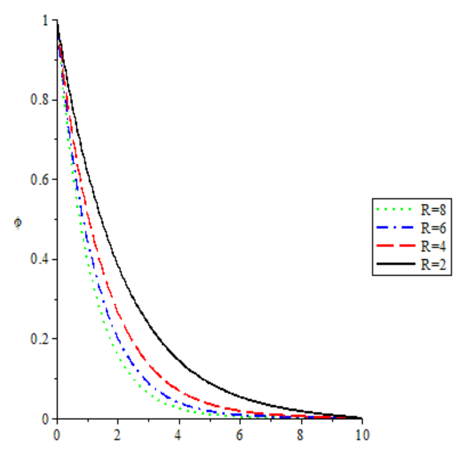

Figure 3: Concentration Profile for Rc

The nonlinear ordinary differential equations (8) and (9) satisfying boundary conditions (10) have been solved numerically using the Maple 18 for several values of the involved parameters. These are permeability parameter, exponential parameter, and reaction rate parameter of the solute and Schmidt parameter. The interesting features of significant parameters on velocity, concentration, skin friction and Sherwood numbers were noticed, there were marginal correspondence increase/decrease in Skin friction and Sherwood number. Table 1 indicates that variation K which is permeability elicits minimal mass transfer while Tables 2 and 3 indicate increase in mass transfer as Sherwood number and R increase. It was discovered that when all parameters used in the investigations were varied and compared with the existing studies, they showed minimal errors in few variations (Table 4). In essence, the results indicate that Skin friction increases with increasing permeability parameter K. Since K does not appear in the diffusion equation, its effect on mass transfer (Sherwood number) is small. The results also showed that the Sherwood number increased as R *and* Sc increased.

4.0 Conclusion

In conclusion, the study presented the behaviour of the distribution of reactive solute. The solute underwent a first order reaction in steady boundary layer flow in compressible fluid over a stretching surface embedded in porous medium with variable surface concentration. The similarity transformation was used to arrive at a set of ordinary differential equations as obtained from the governing equations. The momentum equation and the equation of concentration were solved numerically using Maple 18. The results showed that the porous or permeability parameter reduced the rate of flow from the wall, while broadening the solute layer. The Schmidt number and the reaction rate parameter reduced the thickness of the solute boundary layer. Most importantly, the effects of initial variable solute distribution over a stretching surface showed that for the every increase in values of K, Sc and R, the concentration reduced correspondingly.

5.0 References

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