Radiative Effect of Dissipation Function in the Development of Temperature Profiles in a Pipe with Isothermal Convection

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Abstract

Temperature profiles in a fluid flow inside a porous vertical channel influenced by the presence of variable thermal conductivity and dissipation function are important features in the study of magneto hydro dynamics. In this study, the effects of temperature and variable fluid properties on unsteady hydro magnetic flow of a radiating gas inside a porous were investigated. The coupled nonlinear partial differential equations derived from the governing equations of the fluid were solved by the method of successive approximation techniques. It was found that the dissipation function was completely neglected at the centre line but maximum at the wall of the channel where a fully developed temperature profiles reached its peak. The heat flow balances at the wall due to high temperature and dissipation function.

Keywords: Thermal conductivity, Magneto hydro dynamics, Temperature profile, Unsteady flow

Abbreviations and Symbols

B – Induced Magnetic Field Vector
H – Applied Magnetic Field Vector
V – Fluid Velocity Vector
q_r – Radiation Flux Vector
C_p – Specific Heat at Constant Pressure
g – Gravitational Acceleration
ℓ - Half Width Channel
P – Pressure
α_R – Roseland Mean Absorption Coefficient
T – Temperature
T_w – Wall Temperature
T_f – Final Temperature $T_f = T_f + \varepsilon$
T_s – Temperature of Static Fluid
θ – Temperature Difference $T^* - T$
K – Thermal Conductivity

U, V, W – Orthogonal Velocity Components
x, y, z, t – Cartesian coordinate and time
M – Hartman Number
P_r – Prandtl number
G_r – Grashof number
R_e – Reynold's number
R_m – Magnetic Reynold's number
α – Thermal diffusivity
β - Volumetric expansion coefficient
μ– Viscosity of fluid
μ' –Magnetic Permeability
ρ – Reference density of fluid
υ – Kinematic viscosity of fluid $\left(\upsilon / \rho \right)$
σ_c – Electrical conductivity of the fluid

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1.0 Introduction

Deformation and flow of materials require mechanical energy which is dissipated and converted to internal energy in form of heat. During the process, dissipative energy flow is converted into internal energy of the material. The velocity, applied magnetic field and dissipation function effect have already been considered in some studies [1-3]. For example, an important work on heat transfer is that of the effect of magnetic field on forced convective heat transfer in parallel plates [4, 5]. This is followed by the heat effect of transverse magnetic field on heat transfer to an electrically and thermally radiating fluid in a parallel plane channel [6] and the influence of electric and magnetic fields on heat transfer to electrically conducting fluid [7].

The radiating effect on thermal instability of a flow in a porous medium between two rotating cylinders has also been studied [8]. Furthermore, the unsteady hydro magnetic flow of a radiating gas in a vertical channel using different approximation for optically thin non-grey gas near equilibrium was also investigated [8]. The effect of dissipation function in the presence of transverse magnetic field on unsteady hydro magnetic flow of a radiating gas, electrically conducted inside a vertical channel was equally studied [9]. Similarly, the effect of thermo physical parameter on electrically conducting fluid inside a porous vertical channel involving dissipative function has been studied [9] with the influence of dissipation function on the temperature profile, usually neglected in some studies [7,10,11]. It was these results that prompted further investigation of the effects of dissipation function and variable thermal conductivity on the temperature profiles of a hydro magnetic flow due to induced magnetic field in a given vertical wall.

2.0 Methods

2.1 Mathematical Formulation and Methods of Solution

The Momentum, Magnetic Fields and Radiation Equations with Dissipation function in terms of Cartesian, Cylindrical and Spherical Coordinates have been provided [2].

$$\nabla.v = 2\mu\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 - \frac{1}{3}(\nabla.v)^2\right] + \mu\left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]^2 + \mu\left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]^2 + \mu\left[\frac{\partial v_x}{\partial z} + \frac{\partial v_2}{\partial x}\right]^2$$

$$\nabla.v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \tag{1}$$

$$\nabla.v = 2\mu\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2 - \frac{1}{3}(\nabla.v)^2\right] + \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]^2 + \mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]^2$$

$$\nabla.v = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial_\theta} + \frac{\partial v_z}{\partial z} \tag{2}$$

$$\tau : \nabla v = 2\mu\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right)^2 - \frac{1}{3}(\nabla.v)^2\right.$$

$$+ \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\theta}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \theta}\right]^2 + \mu\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)^2\right.$$

$$\nabla.v = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} \tag{3}$$

2.2 Physical Problem Model and Governing Equations

Consider the physical problem which consists of an infinite channel formed by parallel plates at a distance ℓ apart where a magnetic field, H, is applied across the ends of the channel, the plates at y = ℓ and y = 0 are maintained at temperature t = 1 and 0 respectively with an electrically conducting fluid under pressure flow through it. The surface temperature is assumed to vary linearly along the vertical direction taken as x – axis. The physical configuration of the problem is sketched in Figure 1.

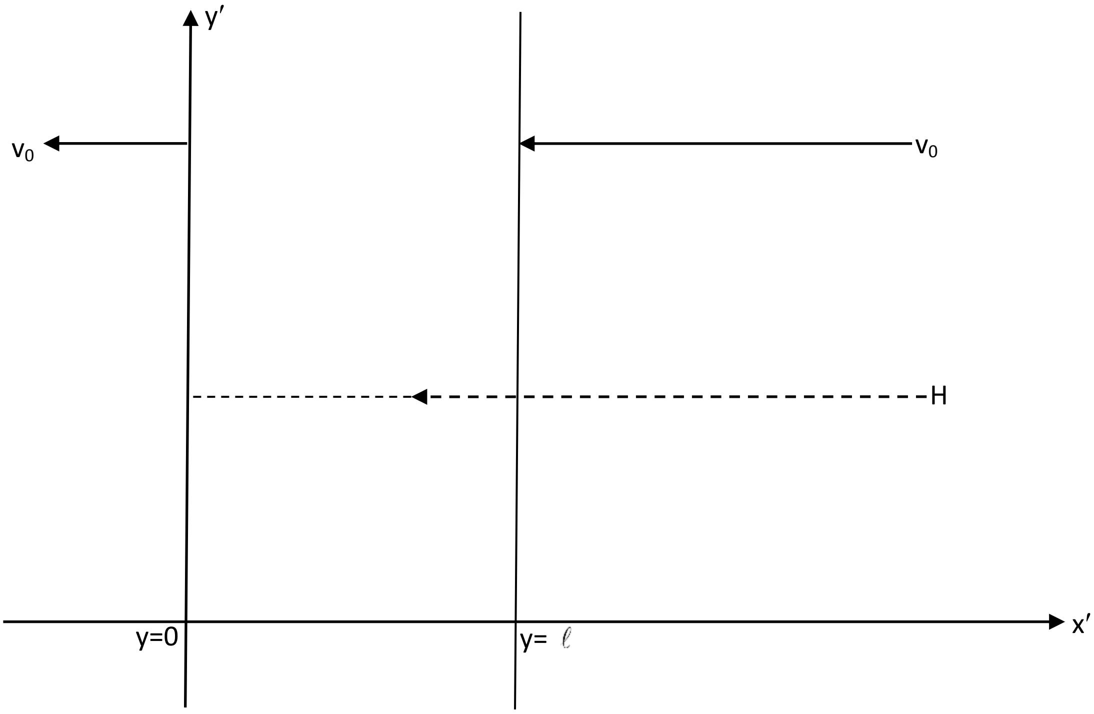

Fig. 1: Physical Model and Coordinate System

The channel walls are assumed electrically non-conducting, where the magnetic field is zero at the plate, y = ℓ and y = 0 for fully developed laminar flow, the velocity and induced magnetic field have only a vertical component and all of the physical variables are function of time (t) except temperature and pressure. Equations (4) to (6) are the governing equations of momentum, magnetic and dissipation function along the y – axis;

$$\ell_s\left(\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'}\right) = \mu\frac{\partial^2 u'}{\partial y'} + H_0\frac{\partial H'\alpha}{\partial y'} + \ell_s g\beta(T - T_2) \tag{4}$$

$$-\frac{\partial H'\alpha}{\partial y^2} = \alpha_0\left[-H_0\frac{\partial u'}{\partial y'} + v_0\frac{\partial H'\alpha}{\partial y'} - \frac{\partial H'\alpha}{\partial t}\right] \tag{5}$$

$$\ell_s c_\rho\left[\frac{\partial^2 t}{\partial t'} - v_0\frac{\partial T'}{\partial y'}\right] = k\frac{\partial T'}{\partial y'^2} - \frac{\partial}{\partial y}\left(-\frac{16\alpha_0 T^3}{3\alpha}\frac{\partial T'}{\partial y'}\right) + \mu_0(\mu\theta + 1)\left(\frac{\partial \mu'}{\partial y'}\right)^2 \tag{6}$$

The equation of continuity is identically satisfied and the viscous dissipation function that is usually neglected had been retained in equation (6) in this model. This study deals with the optically thick limit case, where heat transport in the material is by local radiation and absorption. The transport process is analogous to conduction, and the local net radiation at y is given by

$$q'_y = -\frac{16\sigma c T^3}{3\alpha}\frac{\partial T}{\partial y'} \tag{7}$$

The calculation of temperature fields requires the solution of the dissipation function with radiation effect. The equation is conveniently rewritten in dimensionless form and the generation number, M′, which is the dissipation term in equation 6 is rewritten in simple form;

$$\ell^{c}\rho\frac{DT}{Dt}=k\nabla^{2}T+\frac{1}{2}\eta\dot{\gamma} \tag{8}$$

The fluid is assumed to be purely viscous and of constant viscosity. The thermal properties ℓ, c_{ρ}, k are assumed to be constant. The viscosity has to be specified as a function of temperature; and deformation rate: D/Dt is a substantial time derivative.

$$\frac{\partial T}{\partial t}=\frac{\partial T}{\partial t}+v.\nabla T \tag{9}$$

The equation of energy is made dimensionless by scaling it with the factor $H^{2}/k\Delta T^{0}$

Thus, equation (8) becomes

$$\frac{\ell^{c}\rho vH}{k}\frac{H}{L}\frac{DT^{*}}{Dt^{*}} \tag{10}$$

$$M'=H_{\alpha}\left(\frac{L}{\mu v_{0}}\right)^{1/2} \tag{11}$$

Great N_{GZ} number

$$N_{GZ}=\nabla_{*}T^{*}+\frac{1}{2}\frac{V^{2}\eta^{0}}{k\nabla T^{0}}\eta^{*}\left(\dot{\gamma}^{*};\dot{\gamma}^{*}\right) \tag{12}$$

The physical model is constructed in such a way that

$$\underline{V}=\left(u',-v_{0},0\right), H=\left(H'_{\alpha},H_{0},0\right)\ \underline{q}r=\left(0,q'_{y},0\right) \tag{13}$$

where, $\underline{V}$ is the velocity of the fluid, H is the magnetic field and $\underline{q}_{r}$ is the heat flux. The electromagnetic unit has been employed so that the parameter μ of free space is unity such that.

$$B=\mu\mathrm{H}=\mathrm{H} \tag{14}$$

Equation of continuity: $$\frac{\partial u'}{\partial x'}=0 \tag{15}$$

Momentum Transport Equations:

$$\ell_{s}\left(\frac{\partial u'}{\partial t'}-\frac{\partial u'}{\partial y'}\right)=\mu\frac{\partial^{2}u'}{\partial y'}+H_{0}\frac{\partial H'_{\alpha}}{\partial y'}+\ell_{s}g\beta\left(T-T_{s}\right) \tag{16}$$

Applied Magnetic Equations:

$$-\frac{\partial^2 H_\alpha}{\partial y'^2}=\sigma_c\left[H_0\frac{\partial u'}{\partial y'}+v_0\frac{\partial^2 H'_\alpha}{\partial y'}-\frac{\partial^2 H'_\alpha}{\partial t'}\right] \tag{17}$$

Energy Transport Equation:

$$\ell_s C_\rho\left[\frac{\partial T}{\partial t'}-v_0\frac{\partial T}{\partial y'}\right]=\left[\alpha k_0\frac{\partial\theta}{\partial y}\right]\frac{\partial T}{\partial y'}+k_0(\alpha\theta+1)\frac{\partial^2 T'}{\partial y'^2}+\frac{\partial}{\partial y}\left(\frac{16\theta cT^3}{3\alpha}\frac{\partial T'}{\partial y'}\right)+\mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{18}$$

where the expression for q'_y in equation (7) has been made use of.

Equations (3) to (6) are solved subject to non-dimensional quantities boundary conditions:

$$\left.\begin{array}{l} u=H=0,\ \theta=0, t=0 \\ u=H=1, \theta=1, t=1 \end{array}\right\} \tag{19}$$

2.3 Solution Using Successive Approximation Techniques

The following equations are solved using the successive approximation techniques;

$$\mathrm{Re}\left[\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}\right]=\frac{\partial u}{\partial t}+M^2\frac{\partial H}{\partial y}+Gr(\theta+1) \tag{20}$$

$$-R_m\frac{\partial u}{\partial y}=\frac{\partial^2 H}{\partial y^2}+R_m\frac{\partial H}{\partial y}-R_m\frac{\partial H}{\partial t} \tag{21}$$

$$\mathrm{Re}\,p_r\frac{\partial\theta}{\partial t}=(1+A_7)\frac{\partial^2\theta}{\partial y^2}+\mathrm{Re}\,p_r\frac{\partial\theta}{\partial y}+A_1\theta\left(\frac{\partial\theta}{\partial y}\right)^2+A_2\theta\left(\frac{\partial\theta}{\partial y}\right)^2+A_3\left(\frac{\partial\theta}{\partial y}\right)^2$$

$$+\alpha\left(\frac{\partial\theta}{\partial y}\right)^2+A_4\theta^3\frac{\partial^2\theta}{\partial y^2}+A_5\frac{\partial^2\theta}{\partial y^2}+A_6\theta\frac{\partial^2\theta}{\partial y^2} \tag{22}$$

Equations (20) to (22) were used to obtain θ_0. In doing this, the non- linear terms in equation (22) was ignored. From equation (22), θ_0 was obtained as [4]

$$\theta_0(y,t)=\lambda\,\ell^{at}\left(\ell^{\phi_1 y}-\ell^{-\phi_2 y}\right) \tag{23}$$

where

$$\lambda=\frac{\ell^{-a}}{\ell^{\phi_1}-\ell^{-\phi_1}} \tag{24}$$

The magnetic field, H_0 was obtained by re-arranging of equation (21), using approximate analytical techniques

$$H_0(t,y)=\lambda,\ell^{bt}\left(\ell^{\phi_1 y}-\ell^{-\phi_1 y}\right) \tag{25}$$

where

$$\lambda_1=\frac{\ell^{-b}}{\ell^{\phi_t}-\ell^{-\phi_1}} \tag{26}$$

$$H_0(y,t) = A_4\, e^{kt+\phi_5 y} + A_5 e^{kt+\phi_6 y} + W_1 e^{rt+ky} + W_2 e^{at-ky} + W_2 e^{at+\phi_1 y} + W_4 e^{at-\phi_2 y} \quad (27)$$

where

$$A_1 = \frac{16\sigma c N^2}{\alpha k_0}, A_2 = \frac{32\sigma c N T_i}{\alpha k_0}, A_3 = \frac{16\sigma c T_i^3}{\alpha k_0}, A_4 = \frac{16\sigma c N^3}{3\alpha k_0},$$

$$A_5 = \frac{16\sigma c N^2 T_i}{\alpha k_0}, A_7 = \frac{16\sigma c T_i^3}{3\alpha k_0}, \quad M' = \frac{V_0 \mu_0}{N k_0}$$

$$\partial_1 \frac{\partial_2 \theta}{\partial t} = \gamma_2 \frac{\partial_2 \theta}{\partial y^2} + \gamma_1 \frac{\partial \theta}{\partial y} \quad (28)$$

where

$$\gamma_1 = \mathrm{Re}\,\mathrm{Pr} \text{ and } \gamma_2 = (1 + A_7) \quad (29)$$

Having obtained H_0 and θ_0, u_0 was obtained from equation (20) as follows:

$$\mathrm{Re}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \mathrm{Re}\frac{\partial u}{\partial y} + F(y,t,\theta_0,H_0) \quad (30)$$

where

$$F(y,t,\theta_0,H_0) = M^2 \frac{\partial H_0}{\partial y} + Gr(\theta_6 + 1) \quad (31)$$

Solving equation (31) to obtain,

$$F(y,t,\theta_0,H_0) = a_0 \left[\phi \left\{ M^2 \phi_3 F_1 e^{at+\phi_3 y} - M^2 \phi_4 e^{bt-\phi_4 y} + Gr\lambda\, e^{at+\phi_1 y} - Gr\lambda\, e^{at+\phi_2 y} + Gr \right\}\right] \quad (32)$$

$$u_0(y,t) = A^{-n^2 t}\left(Be^{\psi_1 y} + Ce^{\psi_2 y}\right) = Be^{\psi_1 y - n^2 t} + Ce^{\psi_2 y - n^2 t} \quad (33)$$

where

$$\left.\begin{aligned} F_1 &= \frac{e^{-b}}{e^{\phi_3} - e^{-\phi_4}} \\ W_1 &= \frac{e^{-b}}{e^{-\phi_4} - e^{\phi_3}} \end{aligned}\right\} \quad (34)$$

Obtain particular integral of u_0 as

$$u_1 \rho(y,t) = B_1 e^{bt+\phi_3 y} + B_2 e^{bt-\phi_4 y} + B_3 e^{at+\phi_1 \cdot y} + B_4 e^{at-\phi_2 y} + B_5 \quad (35)$$

So that

$$\left.\begin{aligned} B_1 &= \frac{M^2 \phi_3 F_1 a_0}{b - a_0 \phi_3^2 - \phi_1} \\ B_2 &= -\frac{M^2 \phi_4 W_1 a_0}{b_1 - a_0 \phi_4^2 + \phi_4} \\ B_3 &= \frac{Gr\lambda a_0}{a - a_0 \phi_2^2 + \phi_2} \\ B_4 &= \frac{Gr\lambda \sigma_0}{a - a_0 \phi_2^2 + e_2} \end{aligned}\right\} \quad (36)$$

$$Br = Gra$$

Therefore, the complete solution becomes

$$u_0 = Be^{n^2t+\psi_2 y} + Ce^{-n^2t+\psi_2 y} + B_1 e^{bt+\phi_3 y} + B_2 e^{bt-\phi_4 y} + B_3 e^{at-\phi_1 y} + B_4 e^{at-\phi_2 y} + B_5 y \quad (37)$$

To obtain θ_1t needed for developing the temperature profiles influenced by dissipative function, we use equation (23), (27) and (33). In order to do this replace all the non-linear terms in (22) with

$$\theta_1 = \lambda\left(e^{at+\phi_1 y} - e^{at-\phi_2 y} + \theta_\rho(y,t)\right. \quad (38)$$

To get

$$\begin{aligned}\theta_p(y,t) &= C_1 e^{4at+4\phi_1 y} + C_2 e^{4at+(3\phi_1-\phi_2)y} + C_3 e^{4at+(2\phi_1-2\phi_2)y} + C_4 e^{4at+(\phi_1-3\phi_2)y} + C_5 e^{4at+4\phi_2 y} \\ &+ C_6 e^{3at+3\phi_1 y} + C_7 e^{3at+(2\phi_1-\phi_2)y} + C_8 e^{3at+(\phi_1-2\phi_2)y} + C_9 e^{3at-3\phi_2 y} + C_{10} e^{2at+2\phi_1 y} + C_{11} e^{2at+(\phi_1-\phi_2)y} \\ &+ C_{12} e^{2at-2\phi_2 y} + C_{13} e^{-2n^2t+2\psi_1 y} + C_{14} e^{-2n^2t+2\psi_2 y} + C_{15} e^{2bt+2\phi_3 y} + C_{16} e^{2bt-2\phi_4 y} + C_{17} e^{2at+2\phi_1 y} \\ &+ C_{18} e^{2at-2\phi_2 y} + C_{19} + C_{20} e^{-2n^2t+(\psi_1+\psi_2)y} + C_{21} e^{(n^2t+b)t+(\psi_1+\phi_2)y} + C_{22} e^{(n^2+b)t+(\psi_1+\phi_3)y} \\ &+ C_{23} e^{(-n^2+b)t+(\psi_1+\phi_4)y} + C_{24} e^{2bt+(\phi_3-\phi_4)y} + C_{25} e^{(-n^2+b)t+(\psi_2-\phi_4)y} + C_{26} e^{2at+(\phi_1-\phi_2)y} + C_{27} e^{at-\phi_2 y} \\ &+ C_{28} e^{at+\phi_1 y} + C_{29} e^{(-n^2+a)t+(\psi_1+\phi_1)y} + C_{30} e^{(-n^2+a)t+(\psi_1-\phi_2)y} + C_{31} e^{-n^2t+\psi_1 y} + C_{32} e^{(-n^2+a)+(\psi_2+\phi_1)y} \\ &+ C_{33} e^{(-n^2+a)t+(\psi_2-\phi_2)y} + C_{34} e^{n^2t+\psi_2 y} + C_{35} e^{(a+b)t+(\phi_3+\phi_1)y} + C_{36} e^{(a+b)t+(\phi_3-\phi_2)y} + C_{37} e^{bt+\phi_3 y} \\ &+ C_{38} e^{(a+b)t+(\phi_1-\phi 4)y} + C_{39} e^{(a+b)t+(\phi_2+\phi_4)y} + C_{40} e^{bt+\phi_4 y} \qquad (39)\end{aligned}$$

Where, ci = i = 1, 2, are obtained from equation (39) as follows;

$$C_1 = \frac{A_1\lambda^4\phi_1}{16\phi_1^2 + \gamma\phi_1 - \gamma_1 4a}, \quad C_2 = \frac{A_1\lambda^4(3\phi_1-\phi_2)}{(3\phi_1-\phi_2)^2 + \gamma(3\phi_1-\phi_2) - \gamma_1 4_a}, \quad C_3 = \frac{A_1\lambda^4(2\phi_1-2\phi_2)}{(2\phi_1-2\phi_2)^2 + \gamma(\phi_1-2\phi_2) - \gamma_1 4a}$$

$$C_4 = \frac{A_1\lambda^4(\phi_1-3\phi_2)}{(\phi_1-3\phi_2)^2 + \gamma_2(\phi_1-3\phi_2) - \partial_1 4a}, \quad C_5 = \frac{A_1\lambda^4\phi_2}{16\phi_2^2}, \quad C_6 = \frac{A_2\lambda^4\phi_1}{9\phi_1^2 + \gamma_2 3\phi_1 - \gamma_1 3a}$$

$$C_7 = \frac{-A_2\lambda^4(2\phi_1-\phi_2)}{(2\phi_1-\phi_2)^2 + \gamma_2(2\phi_1-\phi_2) - \partial_1 3a}, \quad C_8 = \frac{A_1\lambda^4(\phi_1-3\phi_2)}{(\phi_1-3\phi_2)^2 + \gamma(\phi_1-3\phi_2) - \gamma_1 4a}, \quad C_9 = \frac{A_2\lambda^3\phi_2}{9\phi_2^2 - \gamma_1 3\phi_2 - \gamma_1 3a}$$

$$C_{10} = \frac{A_3\lambda^2\phi_1 + 2\alpha\lambda^2}{4\phi_1^2 + \gamma_2 2\phi_1 + \gamma_1 2a}, \quad C_{11} = \frac{-a\lambda^2(\phi_1-\phi_2)}{(\phi_1-\phi_2) + \gamma_2(\phi_1-\phi_2) - \gamma_1 2a}, \quad C_{12} = \frac{2a\lambda^2\phi_1^2 - A_3\lambda^2\phi_2}{4\phi_2^2 + \gamma_2(2\phi_2) + \gamma_1 2a}$$

$$C_{13} = \frac{M'B^2\psi_1^2}{4\psi_1^2 + \gamma_2^2\phi_1 - \gamma_1 2b}, \quad C_{14} = \frac{M'C\psi_2^2}{4\psi_2^2 + \gamma_2\psi_2 - \gamma_1 2n^2}, \quad C_{15} = \frac{M'B_1^2\phi_3^2}{4\phi_3^2 + \gamma_2^2\phi_3 - \gamma_1 2b} A$$

$$C_{16} = \frac{M'B_2^2\phi_4^2}{4\phi_4^2 + \gamma_2\phi_4 - \gamma_1 2b}, \quad C_{17} = \frac{M^2 B_3\phi_1^2}{4\phi_1^2 + \gamma_2 2\phi_1 - \gamma_1 2a}, \quad C_{18} = \frac{M'B_4^2\phi_2^2}{4\phi_2^2 + \gamma 2\phi_2^2 + \gamma_1 2a}, \quad C_{19} = M'B_5^2$$

$$C_{20} = \frac{2MB_4^2\phi_2^2}{4\phi_2^2 + \gamma_2\phi_2 - \gamma_1 2n^2}, \quad C_{21} = \frac{2M'BB_1\phi_1\phi_3}{(\psi_1+\phi_3)^2 + \gamma(\psi_1+\phi_3) - \gamma_1 n}, \quad C_{22} = \frac{-2M'BB_2\psi_1\phi_4}{(\psi_1-\phi_3)^2 + \gamma(\psi_1+\phi_3) - \gamma_1 n}$$

$$C_{23} = \frac{2M'B_1B_2\phi_3\phi_4}{(\phi_3-\phi_4)^2 + \gamma(\phi_3-\phi_4)^2 + \gamma_1 2b}, \quad C_{24} = \frac{M'2B_1\psi_1\phi_1}{(\psi_2+\phi_3)^2 + \gamma(\psi_1+\phi_3) - \gamma_1(b-n^2)}$$

$$C_{25}=\frac{-2M'C\psi_3\phi_1}{(\psi_2-\phi_4)^2+\gamma(\psi_2-\phi_4)-\gamma_1(b-n^2)},\quad C_{26}=\frac{-2M'B_3B_4\phi_1\phi_2}{(\phi_1-\phi_2)^2+\gamma_2(\phi_1-\phi_2)+\gamma_1 2a},\quad C_{27}=\frac{-2M'B_4B_5\phi_2}{\phi_2^2+\gamma_2\phi_2-\gamma_1 a}$$

$$C_{28}=\frac{2M'B_3B_5\phi_1}{\phi_1^2+\gamma\phi_1-\gamma_1 a},\quad C_{29}=\frac{-2M'BB_3\psi_1\phi_1}{(\psi_2+\phi_1)^2+\gamma_1(\psi_1-\phi_1)-\gamma_1(a-n^2)},\quad C_{30}=\frac{2M'B_3C\phi_1\psi_2}{(\psi_1-\phi_2)-\gamma_2(\psi_1-\phi_2)\gamma_1(a-n^2)}$$

$$C_{31}=\frac{2M'BB_5\psi_1}{\psi_1^2+\gamma\psi_1+\gamma_1 n^2},\quad C_{32}=\frac{2M'B_3C\phi_1\psi_2}{(\psi_2-\phi_2)^2-\gamma_2(\psi_2+\phi_1)-\gamma_1(a-n^2)},$$

$$C_{33}=\frac{-2M'B_4C\psi_2\phi_2}{(\psi_3-\phi_2)^2+\gamma_2(\psi_2-\phi_2)-\gamma(a-n^2)},\quad C_{34}=\frac{2M'B_5C_2}{\psi_2^2+\gamma_2\psi_2+\gamma_1 n^2},\quad C_{35}=\frac{2M'B_1B_3\phi_1\phi_3}{(\phi_3+\phi_1)^2+\gamma_2(\phi_3+\phi_1)-\gamma_1(a+b)}$$

$$C_{36}=\frac{-2M'B_1B_4\phi_2\phi_3}{(\phi_3-\phi_2)^2+\gamma_2(\phi_3-\phi_2)-\gamma_1(a+b)},\quad C_{37}=\frac{2M'B_1B_5\phi_3}{\phi_3^2+\gamma_2\phi_3-\gamma_1 b},\quad C_{38}=\frac{-2M'B_2B_3\phi_1\phi_4}{(\phi_1-\phi_4)^2+\gamma_2(\phi_1-\phi_5)-\gamma_1(a+b)}$$

$$C_{39}=\frac{2M'B_2B_4\phi_2\phi_4}{\phi_3^2+\gamma_2\phi_3-\gamma_1 b},\quad C_{40}=\frac{-2M'B_2B_5\phi_4}{\phi_4^2+\gamma_2\phi_4-\gamma_1 b}$$

3.0 Results and Discussion

The set of coupled partial differential equation together with the associated boundary conditions have been solved using successive approximation technique. The numerical results were then obtained by simulation to establish the effect of the dissipation function as a result of temperature difference at any specific time (t). In doing this, the values of the following parameters; $R_e = 20$, $P_r = 0.50$, $R_m = 0.05$, Gr = 0.5 and dissipative function, M′ ranging between 1 and 3 were adopted to obtain the approximate values of the temperature by numerical approach for physical realistic solutions (Table 1). The numerical results which describe the temperature profile as a function of dissipation function inside the porous vertical channels is presented in Figure 1. The temperature profiles due to variable parameters; thermal conductivity parameter (α = 0, 1) and dissipative parameter (M' = 0, 1) were adopted to obtain the numerical relationship between variable thermal conductivity (α) and dissipation function (M') of the fluid temperature as presented in Table 2. The dissipation function was zero at the center line and has its maximum value at the wall of the vertical channel, when the temperature in the pipe flow demonstrate the non-uniformity of the viscous dissipation as a result of increase in temperature [2,11,12].

Large radial temperature difference gives rise to conduction of heat towards the wall where a fully developed temperature is reached when the heat flow balances the viscous heat generation. It was also observed that the temperature increases as values of the dissipation function increases at specific given time and when the variable thermal conductivity is neglected, the dissipation function effect increases the temperature to higher values. This implies that the effect of dissipation function and thermal conductivity on unsteady hydro magnetic flow inside the parallel plate has a measureable influence on the fluid inside the porous channel as a result of radiation.

Table 1: Temperature Profiles with Varying Dissipation Function (M') at Specified Time (T)

	t = 0.1	t = 0.2	t = 0.3	t = 0.4	t = 0.5
M′ = 1	.2550892E – 01 .8389408E+ 00 .3629186E+ 01 .1281979E+ 02 .4263970E+ 02	.6128340E– 01 .7508434E+ 00 .3580184E+ 01 .1294958E+ 02 .4340876E+ 02	.1721104E+ 00 .633674E+ 00 .3496833E+ 01 .1304141E+ 02 .4414467E+ 02	.3131268E+ 00 .4798913E+ 00 .3369903E+ 01 .1308403E+ 02 .4483374E+ 02	.4927140E+ 00 .2799398E+ 00 .3187672E+ 01 .1306311E+ 02 .4545843E+ 02
M′ = 2	.4342904E – 01 .1653465E+ 01 .7180083E+ 01 .2539088E+ 02 .8450879E+ 02	.1303282E+ 00 .1476716E+ 01 .7080308E+ 01 .2564490E+ 02 .8603025E+ 02	.3521586E+ 00 .1241812E+ 01 .6911798E+ 01 .2582290E+ 02 .8748509E+ 02	.6343720E+ 00 .9336649E+ 00 .6656087E+ 01 .2590235E+ 02 .8884586E+ 02	.2022308E+ 01 .6502979E+ 00 .5093317E+ 02 .2521757E+ 02 .9195697E+ 02
M′ = 3	.61349076E–01 .2467989E+ 01 .1073098E+ 02 .3796197E+ 02 .1263779E+ 03	.1993730E+ 00 .2202589E+ 01 .1058043E+ 02 .3834022E+ 02 .1286518E+ 03	.5322072E+ 00 .1849948E+ 01 .1032676E+ 02 .3860439E+ 02 .1308255E+ 03	.9556176E+ 00 .1387438E+ 01 .9942272E+ 01 .3872066E+ 02 .1328580E+ 03	.1492948E+ 01 .7863974E+ 01 .9391796E+ 01 .3864605E+ 02 .1346966E+ 03

Table 2: The Relationship Between Dissipation Function (M') And Variable Thermal Conductivity (α)

α = 0 $M^1 = 0$	α = 1 $M^1 = 0$	α = 0 $M^1 = 1$
.7588787E – 02	.76442510E – 02	.2550892E – 01
.2441628E – 01	.2499146E – 01	.3629186 E + 0^0
.7828806E – 01	.8425018E – 01	.3629186E + 0^0
.2487031E + 00	.3105059E + 00	.1281979E + 02
.7706148E + 00	.1491274E + 01	.42639970 + 02

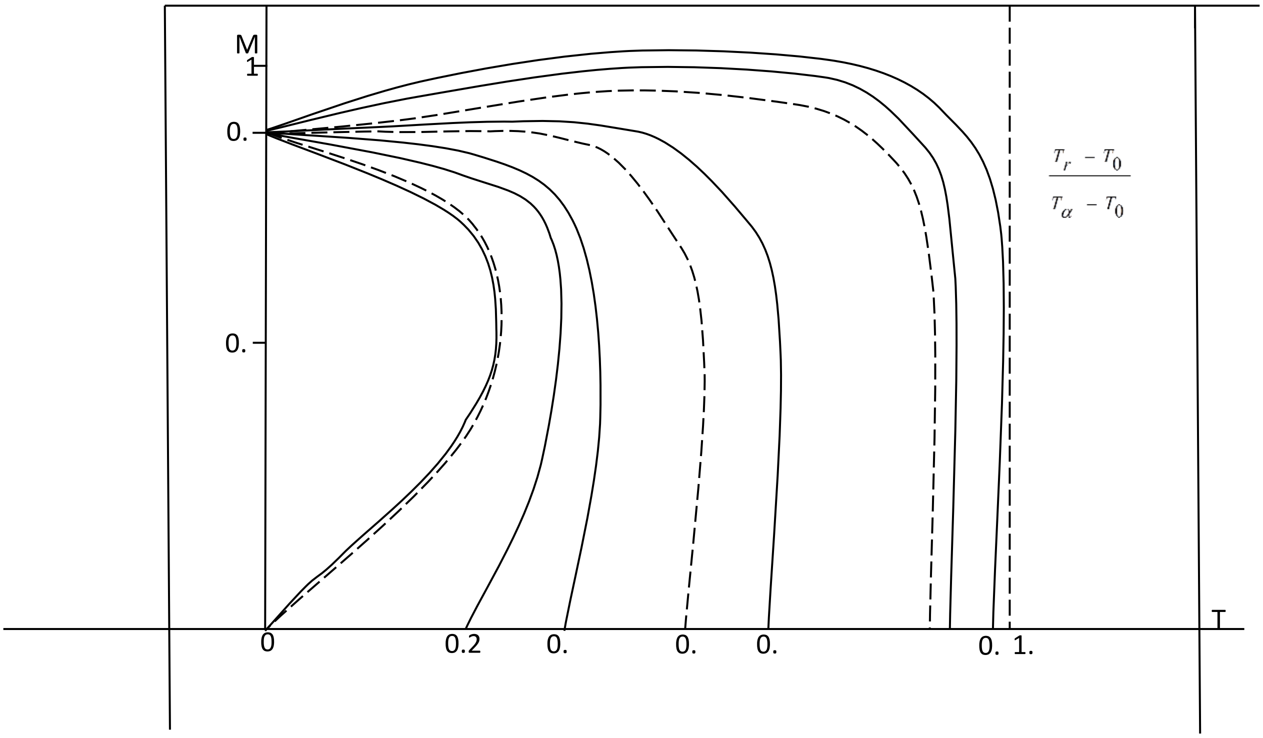

Fig. 2: Dissipation Function Distribution as a Result of Temperature difference

4.0 Conclusion

The temperature profiles associated with the effects of dissipation function and thermal conductivity parameter of a hydro magnetic flow due to an induced magnetic field in a given vertical wall was presented. The distribution indicates the influence of dissipation function on the unsteady hydro magnetic fluid flow as a result of radiation, with high temperature invoked due to the flow current which is placed in a transverse magnetic field. The primary conclusion is that for a combined adverse thermal and dissipation function in the presence of radiative heat transfer, the mode in which the instability sets in is completely governed by the radiating effect.

5.0 References

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